

The process of multiplying the frequency by $3/2$ and then reducing it to within the octave if necessary which generates the Pythagorean tuning of the white keys on the piano can be continued to generate the black keys on the piano. This is called a circle of fifths, which ends up within 1% of the original frequency. The resultant twelve tone scale has the ratio of the frequencies of adjacent notes approximately equal to the twelfth root of 2 (exactly equal with equal temperament rather than Pythagorean tuning). For the piano keyboard as pictured above, the adjacent keys (*i.e.*, keys which are adjacent at the top of the figure, two keys are not adjacent if there is a black key between them) are said to differ by a half step, but if there is a single key in between they differ by a whole step. The white keys are labelled with the first seven letters of the alphabet as depicted. B and C differ by a half step, E and F differ by a half step; C and D differ by a whole step, D and E differ by a whole step. Sharps (\sharp) signify that a note is one-half step higher than the named note, flats signify that a note is one-half step lower than the named note. Thus the black note between D and E can be called either $D\sharp$ or $E\flat$. $B\sharp$ is another name for C , $C\flat$ is another name for B .

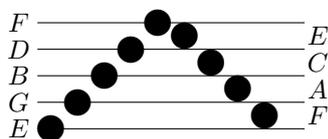
There are occasions where multiple sharps or flats are appropriate. For example $C\sharp\sharp$ is D , $C\flat\flat$ is $B\flat$ (which is the same as $A\sharp$).

Interlude: What are two names for the key between F and G ? Sharping which notes give black keys? Sharping which notes give white keys? Give common names for $E\sharp\sharp\sharp$, for $E\flat\flat$. Give three names for C .

Much of the “sound” of a piece of music is determined by the scale (also called mode) in which it is written. We shall consider only seven note scales (or modes). Scales are named for the dominant or home note in a composition (often the first and last note played), and the pattern of half and whole steps which separate it from the other notes. For example, the white keys on the piano starting with C provide the C major scale: they start on C and follow the sequence of gaps to the next note $W\text{W}H\text{W}\text{W}H$ until the octave is reached (W designates whole step, H designates half step). The sequence of white keys starting with A is the A minor scale: it starts with A and follows the sequence $W\text{H}\text{W}\text{W}H\text{W}\text{W}$.

If one wants the A major scale, One starts with A , but in order to achieve the pattern $W\text{W}H\text{W}\text{W}H$ one must use $C\sharp$ in place of C , then D gives the needed half step, hence E a whole step. But but $F\sharp$ and $G\sharp$ are needed to provide two more whole steps, which leaves a half step to A . the C minor scale is obtained using $E\flat$, $A\flat$, $B\flat$.

So that music can be written on the standard staff, it is necessary to use each of the first seven letters of the alphabet once (modified by sharps and flats as needed) for a seven note scale.



Another manifestation of patterns in music is the repetition/modification of sequences of notes. These include repetition (repeating the same sequence of notes), inversion (vertical reflection across a horizontal axis), retrogression (horizontal reflection across a vertical axis), translation (vertical shift), and cycling (changing the starting note). Examples of these for the sequence $ACBDG$ include repetition: $ACBDG$; inversion: $ECDBF$ or $AFGEB$ or ... ; retrogression: $GDBCA$; translation: $BDCEA$ or $DFEGC$ or ... ; cycling: $CBDGA$ or $DGACB$ or These alterations can also be combined such as retrograde inversion (retrogression and inversion): $FB\text{DCE}$ or $BEGFA$ or ... ; translation and inversion: $FDECG$ or $FDECG$ or ... [this is just an inversion] ; cycle and retrogression: $DBCAG$ or $CAGDB$ or ... ; cycle and inversion: $CDBFE$ or $EBAFG$ or ... ; translation and cycle: $DCEAB$ or $GCDFE$ or These are better seen visually.

repetition

inversion

retrogression

translation

cycling

retrograde inversion

translation and inversion

cycle and retrogression

cycle and inversion

translation and cycle

Exercise: Give an inversion, a retrogression, a translation, and a cycle of ACABGF. Give a retrograde inversion, translation inversion, retrogression cycle, inversion cycle, and translation cycle of that sequence.