Backward iteration in the unit ball

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Analysis/PDE Seminar
Texas A&M University
Outline of my Talk

1. One-dimensional case
   - Forward iteration
   - Backward iteration

2. Multi-dimensional case
   - Preliminaries
   - Main result and examples

3. Conjugations
   - Overview
   - Conjugations near BRFP in the unit ball

4. Parabolic case

5. Future goals
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Forward iteration

Let $f$ be analytic self-map of $\mathbb{D} = \{ z : |z| < 1 \}$

$n$-th iterate of $f$ $f_n = f \circ \ldots \circ f$

$n$ times

By Schwarz’s lemma, $f$ is a contraction in the pseudo-hyperbolic metric

$$d(z, w) = \left| \frac{z - w}{1 - \overline{w}z} \right|$$

Theorem (Denjoy-Wolff)

If a self-map of the disk $f$ is not an elliptic automorphism, then there exist a unique point $p \in \overline{\mathbb{D}}$ such that the sequence $f_n(z)$ converges uniformly on compact subsets to $p$.

If $p \in \mathbb{D}$, then $f(p) = p$ and $|f'(p)| < 1$

If $p \in \partial \mathbb{D}$, then $f(p) = p$ and $0 < f'(p) \leq 1$ in the sense of non-tangential limits
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If a self-map of the disk \( f \) is not an elliptic automorphism, then there exist a unique point \( p \in \mathbb{D} \) such that the sequence \( f_n(z) \) converges uniformly on compact subsets to \( p \).

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**Theorem (Denjoy-Wolff)**

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If $p \in \mathbb{D}$, then $f(p) = p$ and $|f'(p)| < 1$

If $p \in \partial \mathbb{D}$, then $f(p) = p$ and $0 < f'(p) \leq 1$ in the sense of non-tangential limits
The point $p$ is called the **Denjoy-Wolff point** of $f$.

Cases:
1. $p \in \mathbb{D}$ $f$ is called elliptic
2. $p \in \partial \mathbb{D}$, $f'(p) < 1$ hyperbolic
3. $p \in \partial \mathbb{D}$, $f'(p) = 1$ parabolic
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![Diagram of parabolic point](image)
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If \( p \in \partial \mathbb{D} \), **Julia’s lemma** holds for the point \( p \), and multiplier \( c = f'(p) \leq 1 \):

\[
\forall R > 0 \quad f (H(p, R)) \subseteq H(p, cR),
\]

where \( H(p, R) \) is a horocycle at \( p \in \partial \mathbb{D} \) of radius \( R \):

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H(p, R) := \left\{ z \in \mathbb{D} : \frac{|p - z|^2}{1 - |z|^2} < R \right\}
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Backward iteration

Backward-iteration sequence: \( \{z_n\}_{n=0}^{\infty}, f(z_{n+1}) = z_n \)

Not always exists: \( f(z) = cz, |c| < 1 \) has no backward iteration sequences.

By Schwarz’s lemma, \( d(z_{n+1}, z_n) \geq d(z_n, z_{n-1}) \) \( \forall n \), so
\( d_n := d(z_{n+1}, z_n) \nearrow. \)

We need additional condition on sequence to converge:

\[ d(z_{n+1}, z_n) \leq a < 1 \quad \forall n \]

(the pseudo-hyperbolic step must be bounded above).
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Theorem (Poggi-Corradini, 2003)

Let \( \{z_n\}_{n=0}^{\infty} \) be a backward-iteration sequence for analytic self-map (not an elliptic automorphism) of the disk \( f \) with bounded pseudo-hyperbolic step \( d(z_n, z_{n+1}) \leq a < 1 \). Then:
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- The sequence converges to the point on the boundary \( q \in \partial \mathbb{D} \), and \( q \) is a fixed point with a well-defined derivative \( f'(q) < \infty \)
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- The sequence converges to the point on the boundary \( q \in \partial D \), and \( q \) is a fixed point with a well-defined derivative \( f'(q) < \infty \).

- If \( q \neq p \), then \( q \) is a **boundary repelling fixed point** (BRFP) (i.e. \( f'(q) > 1 \)). The convergence \( z_n \rightarrow q \) is non-tangential.
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- If \( q \neq p \), then \( q \) is a boundary repelling fixed point (BRFP) (i.e. \( f'(q) > 1 \)). The convergence \( z_n \to q \) is non-tangential.

- If \( q = p \), then \( z_n \to q \) tangentially. It may happen only in parabolic case.
Multi-dimensional case

\[ \mathbb{C}^N, \text{inner product} (Z, W) = \sum_{j=1}^{N} Z_j \overline{W}_j, \quad \|Z\|^2 = (Z, Z) \]

Unit ball \( \mathbb{B}^N = \{ Z \in \mathbb{C}^N : \|Z\| < 1 \} \)

Julia’s lemma in \( \mathbb{B}^N \)

Let \( f \) be a holomorphic self-map of \( \mathbb{B}^N \) and \( X \in \partial \mathbb{B}^N \) such that

\[ \lim \inf_{Z \to X} \frac{1 - \|f(Z)\|}{1 - \|Z\|} = \alpha < \infty \]

Then there exists a unique \( Y \in \partial \mathbb{B}^N \) such that \( \forall R > 0 \)

\[ f(H(X, R)) \subset H(Y, \alpha R). \]
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Horosphere of center \( X \in \partial B^N \) and radius \( R > 0 \):
\[
H(X, R) = \left\{ Z \in B^N : \frac{|1 - (Z, X)|^2}{1 - \|Z\|^2} < R \right\}
\]

Multi-dimensional version of Denjoy-Wolff theorem holds:

**Theorem (MacCluer, 1983)**

If \( f \) has no fixed points in \( B^N \), then \( f^n \) converges uniformly on compacta to \( p \in \partial B^N \), the number \( c := \liminf_{Z \to p} \frac{1 - \|f(Z)\|}{1 - \|Z\|} \in (0, 1] \) is a multiplier of \( f \) at \( p \).

*\( f \) is called **hyperbolic** if \( c < 1 \) and **parabolic** if \( c = 1 \).*

We will call \( f \) **elliptic** if it has unique fixed point inside of the ball (WLOG fixed point is 0) and \( f \) is not unitary of any slice (i.e. with \( \|f(Z)\| < \|Z\| \ \forall Z \in B^N \backslash \{0\} \)).
Horosphere of center $X \in \partial \mathbb{B}^N$ and radius $R > 0$:

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Siegel domain: $\mathbb{H}^N = \{(z, w) \in \mathbb{C} \times \mathbb{C}^{N-1} : \text{Re}z > \|w\|^2\}$

is biholomorphically equivalent to the unit ball $\mathbb{B}^N$ via Cayley transform: $C : \mathbb{B}^N \rightarrow \mathbb{H}^N$

$C((z, w)) = \left(\frac{1+z}{1-z}, \frac{w}{1-z}\right)$ \quad $C^{-1}((z, w)) = \left(\frac{z-1}{z+1}, \frac{2w}{z+1}\right)$
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Crucial difference between $\mathbb{D}$ and $\mathbb{B}^N$: all results and estimates are weaker in orthogonal dimensions.
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- Pseudo-hyperbolic disk is a Euclidean disk.
- Pseudohyperbolic ball is a Euclidean ellipsoid with $R > r$ and $\frac{R}{r} \to \infty$ as $z \to \partial \mathbb{B}^N$. 

Olena Ostapyuk (K-State)
Theorem 1. (O —, 2010)

Let $f$ be a analytic self-map of $\mathbb{B}^N$ of hyperbolic or elliptic type, $\{Z_n\}$ be a backward-iteration sequence with bounded pseudo-hyperbolic step $d_{\mathbb{B}^N}(Z_n, Z_{n+1}) \leq a < 1$. Then:
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1. There exists a point $q$ on the boundary of the ball (different from the Denjoy-Wolff point) such that $Z_n \xrightarrow[n \to \infty]{} q$. 
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2. $\{Z_n\}$ stays in a Koranyi region with vertex $q$ (Koranyi regions are weaker analogs of non-tangential regions in higher dimension).
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3. Julia's lemma holds for $q$ with multiplier $\alpha \geq \frac{1}{c} > 1$, i.e. $f(H(q, R)) \subset H(q, \alpha R) \forall R > 0$. 
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Definition

A point \( q \in \partial \mathbb{B}^N \) is called a boundary repelling fixed point if Julia’s lemma holds for \( q \) with multiplier \( \alpha > 1 \).
Idea of the proof in hyperbolic case:

\[ t_n := \text{Re} \ z_n - \|w_n\|^2 \sim c^n \text{ (by Julia’s lemma)} \]

\[ \|pr(Z_n) - pr(Z_{n+1})\| \leq C \sqrt{t_n} \sim c^{n/2} \]
Idea of the proof in hyperbolic case:

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In elliptic case we need the following

**Lemma**

Let \( f \) be a self-map of the unit ball \( \mathbb{B}^N \) fixing zero, not unitary on any slice. Fix \( r_0 > 0 \), define \( M(r) := \max \| f(r\mathbb{B}^N) \|, \ r \in [r_0, 1) \). Then there exists \( c < 1 \) such that

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\frac{1 - r}{1 - M(r)} \leq c \quad \forall r \in [r_0, 1)
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$$\frac{1 - r}{1 - M(r)} \leq c \quad \forall r \in [r_0, 1)$$
Idea of the proof in elliptic case:

\[ t_n := 1 - \|Z_n\| \sim c^n \text{ (by lemma)} \]

\[ \phi_n := \text{arc-length}\left(\frac{Z_n}{\|Z_n\|}, \frac{Z_{n+1}}{\|Z_{n+1}\|}\right) \sim \sqrt{t_n} \sim c^{n/2} \]
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Idea of the proof in elliptic case:

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\[ \phi_n := \text{arc-length} \left( \frac{Z_n}{\|Z_n\|}, \frac{Z_{n+1}}{\|Z_{n+1}\|} \right) \sim \sqrt{t_n} \sim c^{n/2} \]
A BRFP with multiplier $\alpha$ is called isolated if it has a neighborhood with no other BRFPs with multiplier $\leq \alpha$.

In the hyperbolic and elliptic cases we have the following Characterization of BRFP in terms of backward-iteration sequences:

Every backward-iteration sequence with bounded hyperbolic step converges to a BRFP; and if BRFP is isolated, then we can construct a backward-iteration sequence with bounded hyperbolic step that converges to it.

In 1-dimensional case all boundary fixed points are isolated (corollary of the theorem of Cowen and Pommerenke, 1982), so the above characterization is "if and only if".
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Example 1. (O —, 2010):

\[ f : \mathbb{H}^2 \to \mathbb{H}^2, \quad f(z, w) = (2z + w^2, w), \] hyperbolic with multiplier 1/2 at the Denjoy-Wolff point \( \infty \)

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**Definition**

We will call the union of all backward iteration sequences with bounded step tending to a BRFP $q$ a **stable set at** $q$.

The stable set at each BRFP $(r, ir^2)$ in the Example 1 is

$$\{(z, r) \mid \text{Re} z > r^2\}$$

and has dimension 1.

**Conjecture**

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(Semi) conjugations

**Goal:**

For self-map $f$ of $\mathbb{D}$ (or $\mathbb{B}^N$), solve an equation

$$\psi \circ f = \eta_f \circ \psi,$$

where $\psi : \mathbb{D} \to \Omega$ (resp. $\psi : \mathbb{B}^N \to \Omega$) is unknown holomorphic function to a complex manifold $\Omega$, and $\eta_f$ is a simple map (e.g. biholomorphism) of $\Omega$. 

Olena Ostapyuk (K-State)
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\[\begin{array}{ccc}
\mathbb{D} & \xrightarrow{f} & \mathbb{D} \\
\downarrow \psi & & \downarrow \psi \\
\Omega & \xrightarrow{\eta_f} & \Omega
\end{array}\]
If $f$ is elliptic with $f'(p) \neq 0$, then

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\psi \circ f = f'(p) \cdot \psi
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with $\psi : \mathbb{D} \rightarrow \mathbb{C}$. 
**Koenigs, 1884**

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**Böttcher, 1904**

*If* $f$ *is elliptic with* $f'(p) = 0$, *then*

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\psi \circ f = \psi^n
$$

*with* $\psi$ *defined in a neighborhood of* $p$. 
<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Condition</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Koenigs</td>
<td>1884</td>
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<td>Valiron</td>
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If $f$ is \textit{parabolic}, then
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with $\psi : \mathbb{D} \to \mathbb{H}$ (\textit{non-zero step case}) or $\psi : \mathbb{D} \to \mathbb{C}$ (\textit{zero step case}).
Pommerenke, Baker and Pommerenke, 1979

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Poggi-Corradini, 2000 (backward iteration):

An analytic self-map of the unit disc \( \mathbb{D} \) \( f \) with BRFP \( 1 \in \partial\mathbb{D} \) and multiplier \( \alpha \) at 1 can be conjugated to the automorphism \( \eta(z) = (z - a)/(1 - az) \), where \( a = (\alpha - 1)/(\alpha + 1) \):

\[
\psi \circ \eta(z) = f \circ \psi(z),
\]

via an analytic map \( \psi \) of \( \mathbb{D} \) with \( \psi(\mathbb{D}) \subseteq \mathbb{D} \), which has non-tangential limit 1 at 1.
Conjugations in several dimensions

Bracci, Gentili, Poggi-Corradini, 2010; hyperbolic case

Let $f : \mathbb{B}^N \rightarrow \mathbb{B}^N$ be a hyperbolic analytic self-map with Denjoy-Wolff point $p \in \partial \mathbb{B}^N$ and multiplier $c < 1$. If

1. There exists special sequence $f_n(Z_0) \rightarrow p$ and
2. the $K$-limit $\lim_{Z \rightarrow p} \frac{1 - \langle f(Z), p \rangle}{1 - \langle Z, p \rangle}$ exists,

then there is a non-constant analytic function $\psi : \mathbb{B}^N \rightarrow \mathbb{H}$ such that

$$\psi \circ f = \frac{1}{c} \cdot \psi$$
Theorem 2. (O —, 2009) (N-dimensional case, backward iteration)

Suppose \( f : \mathbb{H}^N \to \mathbb{H}^N \) is an analytic function and 0 is an isolated boundary repelling fixed point for \( f \) with multiplier \( 1 < \alpha < \infty \). Then \( f \) is conjugated to the automorphism \( \eta(z, w) = (\alpha z, \sqrt{\alpha} w) \)

\[ \psi \circ \eta(Z) = f \circ \psi(Z), \]

via an analytic intertwining map \( \psi \).

Construction of \( \psi \):

\[ \psi = \lim_{n \to \infty} \{ f_n \circ \tau_n \circ p_1 \} \]

where \( p_1(z, w) := (z, 0) \) is the projection on the first (radial) dimension, so

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and is essentially one-dimensional map.
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The image of $\psi$ in $\mathbb{H}^N$:
Corollary

Since image of $\psi$ is always a subset of stable set, the dimension of stable set is at least 1.
Theorem 3. (O —, 2009)

Under some regularity condition, it is possible to improve $\psi$ such that

$$\psi(z, w) = \psi(p_L(z, w)),$$

where $p_L$ is a projection on the first $L$ dimensions.

Condition is

$$f(z, w) = (\alpha z + o(|z|), Aw + o(|z|^{1/2}))$$

e.g. $A = \text{Diag}(\sqrt{\alpha}, \ldots \sqrt{\alpha}, \beta_1, \ldots \beta_{N-L})$, where $\beta_j < \sqrt{\alpha}$
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**Parabolic case in the disk**

Since $d(z_n, z_{n+1}) \leq d(z_{n-1}, z_n)$, pseudo-hyperbolic step $d_n := d(z_n, z_{n+1})$ must have limit: $d_n \xrightarrow{n \to \infty} b$

Subcases (do not depend on the choice of sequence):

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- **non-zero step**
  - tangentially

- **zero step**
  - radially

- **other: not known**
Parabolic case in the ball: Zero and non-zero step cases are defined only for sequences.

Open question

Is it true that if \( d_{B^N}(f_n(Z_0), f_{n+1}(Z_0)) \to 0 \) for some \( Z_0 \in B^N \), then \( d_{B^N}(f_n(Z), f_{n+1}(Z)) \to 0 \) for all \( Z \in B^N \)?

Claim

If the sequence of forward iterates \( \{Z_n\}_{n=1}^{\infty} \) for parabolic self-map of the unit ball is restricted, then it must have zero step, i.e., \( d_{B^N}(Z_n, Z_{n+1}) \to 0 \). In particular, non-zero-step sequence cannot converge non-tangentially.

The only known parabolic examples in \( \mathbb{H}^N \) are:

- Automorphisms (translations):
  
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Given one-dimensional $\phi : \mathbb{H} \to \mathbb{H}$ of hyperbolic or parabolic type, with the Denjoy-Wolff point $\infty$ and BRFP $iy_0$,

construct $f(z, w) := (\phi(z - w^2) + w^2, w)$. Then:

- $f$ is the self-map of $\mathbb{H}^2$ with the Denjoy-Wolff point $\infty$ and has the same type and same multiplier at $\infty$ as $\phi$.
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Thank you!

http://arxiv.org/abs/0910.5451