The most important thing in social dance is to end up with the partner you started with. We shall illustrate this with square dance by tracking the movement of the women, assuming the men do not move. This is readily modified to study contra dance, where both men and women move within sets of four (two couples), but at the end of a musical sequence the couples have exchanged places. Extending it to squares with both men and women moving, or Scottish reels with four couples in a row, or ... is conceptually the same, but may be more difficult because more bodies are involved (however, in some cases it s essentially the same as just four bodies).

We shall number the positions in a square as 1 for the head couple (nearest the music), and 2, 3, and 4 proceeding counterclockwise around the square so that positions 1 and 3 are opposite each other, as are positions 2 and 4. Dance figures result in the dancers exchanging positions, for example, head ladies chain results in the ladies in positions 1 an 3 exchanging positions, all four ladies chain results in ladies 1 an 3 exchanging positions and ladies 2 and 4 exchanging positions. One could also have ladies 1 and 2 exchanging positions, or the ladies in positions 1, 2, and 3 progressing to positions 2, 3, and 1, respectively (it is common in Scottish set dancing to have only three of the four couples dancing at a time).

The result of dancing a figure could be summarized as where the women end up, e.g., all four ladies chain which exchanges the women in positions 1 and 3 and the women in positions 2 and 4 could be summarized as 1234 becomes 3412. However, since each position starts and ends with a woman in it, a permutation has occurred, and we summarize movements as cycles of changing positions. In cycle notation four ladies chain is (13)(24) because the woman in position 1 moves to position 3 and the woman in position 3 moves to position 1; and the woman in position 2 moves to position 4 and the woman in position 4 moves to position 2. If each woman moved to their left (clockwise) there would be a single cycle (1432) indicating that the woman in position 1 moves to position 4, the woman in position 4 moves to position 3, the woman in position 3 moves to position 2, and the woman in position 2 moves to position 1. Head ladies chain is summarized as (13)(2)(4) indicating that the ladies in positions 1 and 3 exchange places, while the ladies in positions 2 and 4 do not move. Note that we are describing the movement of women in the specified positions and are not identifying the women by their original positions (when the dance began).

Example: What is the result of (13)(2)(4) followed by (123)(4)? The woman in position 1 moves to position 3 by the first permutation, and then the second permutation moves the woman in position 3 to position 1, so the composition of these two permutations leaves the woman originally in position 1 in position 1. The first permutation leaves the woman in position 2 in position 2, and the second permutation moves her to position 3, so the woman originally in position 2 ends up in position 3. The woman originally in position 3 is moved to position 1 by the first permutation, and then to position 2 by the second permutation. Both permutations leave the woman in position 4 unmoved. Hence the net result of the composition of these two permutations is (1)(23)(4).

Exercises: How would you represent all four women staying in place? How would you represent all four women moving one position clockwise? How would you represent all four women moving one position counterclockwise? What is the result of (12)(34) followed by (13)(24)? What is the result of (13)(2)(4) followed by (1234)

The original statement was that we want to get the women back to their original position, which is the problem of inverse mapping or undoing the cycles. If there are only 1- or 2-cycles (i.e., one or two individuals within each set of parentheses), individuals are returned home by the same movement, e.g., (1)(23)(4) composed with (1)(23)(4) is (1)(2)(3)(4). But other movements are more subtle. In order to undo (124)(3), 2 must be returned to 1 to get the woman originally at 1 back home, 4 must be returned to 2 to get the person originally at 2 back home, and 1 must be returned to 4 to get the woman originally at 4 back home. Forming this into a cycle we have (142), and it is readily verified the (124)(3) followed by (142)(3) yields (1)(2)(3)(4) which leaves everybody at home. The inverse of (1423) must send 4 to 1, 2 to 4, 3 to 2, and 1 to 3. This is (1324). Note that these inverses are just the original cycles with the order reversed (I have always listed the cycles starting with the lowest number).

One explanation for the fact that (12)(34) followed by (12)(34) returns everyone home is that with only two numbers within a set of parentheses, reversing the order is the same as leaving the order unchanged ((12) and (21) both interchange positions 1 and 2). Another explanation is that repeating the movement (12) moves each individual one further along a circle of length two, hence back to the original position. Similarly, if (1)(234) is followed by (1)(234) and then by (1)(234) again, the individuals 2, 3, and 4 will each have progressed three positions in their circle of three and be back where they started (of courses individual 1 never moves). *Caveat*: in a set of six individuals, the permutation (123)(45)(6) will need to be repeated 5 times (*i.e.*, performed a total of 6 times) to get everyone back where they started (the underlying concept is least common multiple).

Exercises: Give a movement which is its own inverse, and one which is not its own inverse. If (12)(34) is followed by (14)(23), what movement will return the women to their original position?